# <u>Examiner’s commentary</u>

The essay is an original and challenging application of experimental physics. A typical physics model of cycling relates the required power to five different parameters. In this case the event is cycling up a hill for which the candidate chooses three relevant parameters, power required to overcome air drag, rolling resistance and to gain gravitational potential energy. Equations for each of the powers are derived correctly and efficiently. A few reasonable assumptions are made, consequently the total power needed to ride uphill is a function of the velocity and hill gradient. The velocity is considered as a function of the hill gradient ideally with constant power. The GPS, multi-purpose instrument, is used to determine both the velocity and hill gradient, a power meter, with limited accuracy, to determine the cyclist output and a cyclist in action not a keeping-pace machine. The manipulation of the GPS is complex. The challenges encountered manipulating and making specific measurements are well described. An exponential relationship is proposed, however there is no attempt to support it via linearization or a best-fit equation, a greater effort here would be expected. Overall, the analysis is good with relevant observations about GPS computer accuracy and other relevant factors, though some do lack clarity. The conclusion confirms that the hypothesis is partly incorrect due to the exponential nature mentioned. In the appropriate evaluation, limitations refer to the nature of the hills, GPS, power meter and rider's cadence with their impact on results. Weaknesses are recognized and practical methodological changes for clear improvement proposed, as well as future investigations. Overall an essay with positive achievements, with a completion of the analysis higher results could have been reached.

Investigating the effect of gravity on cyclist's velocity

# Extended Essay

Physics

Word count: 3996

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## INTRODUCTION

As cycling's popularity increases, instruments measuring cycling performance and data they generate become more easily available. Active amateur cyclists increasingly use GPS based cycling computers and even power meters, which were not long ago available only to professional riders. Amateurs improve their training and performance analysing their riding data and comparing them with other riders (amateurs and professionals alike). This in turn allows them to focus in their training or their equipment setup on a particular aspect, which improves their performance most.

This essay focuses on cycling uphill, specifically on how hill gradient effects velocity at certain power output. I have chosen two experimental situations to study the relationship of velocity, hill gradient and power. In first, I gathered riding data myself. I rode various hills with different gradients trying to keep power output constant at 165W.

**Research Question:** How does the hill gradient affect the cyclist's velocity at constant power output? In the experiment, the relationship between the hill gradient and the velocity is investigated.

## AIM

The aim of this investigation is to find out the relationship between the hill gradient and the velocity.

## THEORETICAL FRAMEWORK

### Bicycle Design

To keep a fair test, this section describes the design of the bicycle used in the experiment in order to keep the results controlled. A bicycle consists of three parts: the frame, wheels and drivetrain.

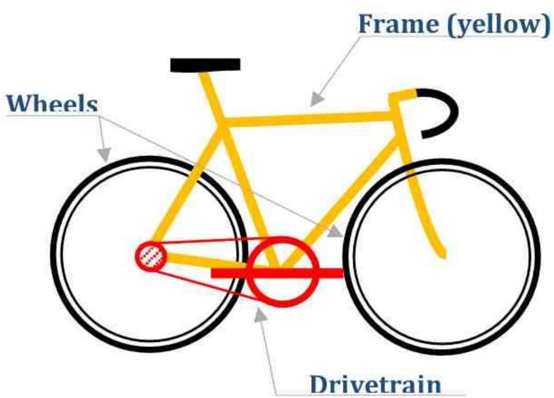


**Figure 1.1 – Diagram of the bicycle used in the experiment.**

Cycling power balance equation

By riding a bicycle, cyclist transforms chemical energy stored in molecules (fats and sugars) into kinetic and potential energy. Some energy is lost due to friction of bicycle components, rolling resistance of wheels and air resistance of the rider and the bicycle.

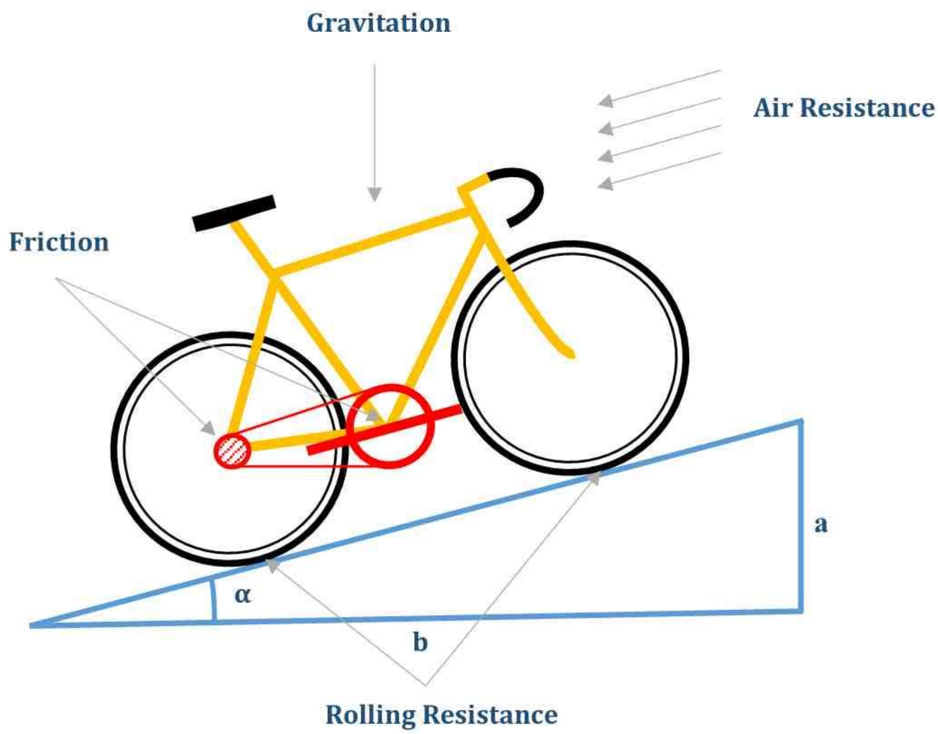


**Figure 1.2 – Diagram of the forces acting on a bicycle on an inclined plane.**

Total power (work over time) needed to move the cyclist and his bike at constant velocity is given by the formula:

$$P_{total} = E_C^{-1}(P_{AD} + P_{RR} + P_{PE})$$

where

$E_C^{-1}$ is drivetrain efficiency coefficient – a factor of how much power is lost due to friction of drivetrain components. Key sources of friction are (i) chain running around front and rear chain rings and through pulley and (ii) bottom bracket. This coefficient is estimated to be 97-98% (Martin, 1998). As this is not the subject of my study, I will simply assume 97% in further calculations as my bicycle is not a top of the range race bike,

$P_{AD}$ is power needed to overcome air resistance also called air drag,

$P_{RR}$ is power needed to overcome rolling resistance of wheels,

$P_{PE}$ is power needed to overcome the gravity or to gain potential energy.

## THE THREE RESISTING FORCES

When investigating the velocity of a cyclist, it is essential to calculate and discuss the three influential forces that affect the rider's cycling performance. In this section, the forces of air-drag, rolling resistance and weight will be analysed with the theories defining them. The friction in the bicycle components is too negligible to be analysed as an impacting aspect to affect the investigation.

### Air-drag

One of the three forces affecting the investigation is air-drag. This force is a function of the size and shape of an object, air density and velocity against wind (not ground) – also called apparent wind velocity. If the wind velocity against ground is zero, then we can substitute ground velocity for apparent wind velocity. To be able to find the specific value of this force and therefore see the effect it might have on the results, the equation that calculates air resisting force is:

$$F_{AD} = \frac{1}{2}\rho C_d A v_a^2$$

where

$\rho$ is air density in $\frac{kg}{m^3}$, which decreases with altitude and temperature,

$C_d$ is air drag coefficient, which is a function of shape and, to lesser extent, of surface material,

A is cross-sectional area in $m^2$ of the rider and the bike perpendicular to the direction of the movement,

$v_a$ is apparent wind velocity or air displacement velocity in $\frac{m}{s}$.

However, during my experiment, where was no wind present to impact the size of the air resisting force. Therefore, at zero wind, the drag force is equal to:

$$F_{AD} = \frac{1}{2}\rho C_d A v_g^2$$

Another important aspect when calculating the size of the air drag is the air density while carrying out the experiment. According to the ideal gas law, the air density is a function of air pressure, temperature and the specific gas constant, which varies with air humidity. Since velocities in my experiments rarely exceed 20 km $h^{-1}$, the effect of air drag is limited. Hence, I am simply using 1.2 kg $m^{-3}$ for my rides around Bratislava (altitude 134 to 527 m), which is a typical value for dry air at given altitudes.

Additionally, the air-drag coefficient (Cd) and cross-sectional area (A) in cycling are another source of influence in the size of the air drag that the cyclist in the investigation is encountering. Air drag coefficient and cross sectional area in cycling are typically studied together and best way to do it is to carry out tests in wind tunnel and compare them with outdoor data. Since, the rider in the experiment has similar characteristics to mine, the value of CdA used in the experiment shall be 0.3 $m^2$.

Rolling Resistance

Another force that has an effect on the investigation is the force caused by rolling resistance. At a flat surface, it is a factor of combined weight of the bike and rider, acceleration due to gravity (perpendicular to the ground) and the rolling resistance coefficient:

$$W = C_{RR} m g$$

where

$C_{RR}$ is coefficient of rolling resistance,

m is the combined mass of the bike and the rider in kilograms,

g is the gravitational acceleration in $ms^{-2}$.

When riding uphill g in not perpendicular to ground, hence for g we need to substitute component of g perpendicular to ground.

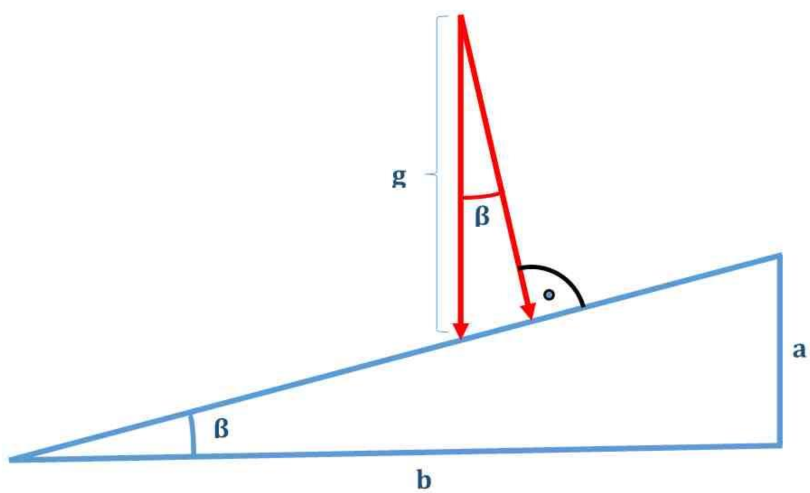


**Figure 1.4 – Diagram of the weight resolved on an inclined plane**

Component of the acceleration due to gravity perpendicular to ground equals:

$$g_p = g \cos \beta$$

where

$g$ is the gravitational acceleration, g = 9.81 $ms^{-2}$.

$\beta$ is the angle of incline.

The steepness of hills is never provided as an angle of incline, but as an incline gradient, which is a ratio of altitude gain over horizontal distance.

$$\tan\beta = \frac{a}{b}$$

$$\beta = \tan^{-1}\left(\frac{a}{b}\right)$$

Rolling resistance force is then:

$$F_{RR} = C_{RR}mg\cos\left(tan^{-1}\left(\frac{a}{b}\right)\right)$$

and consequently, the power needed to overcome rolling resistance is:

$$P_{RR} = C_{RR}mgv_g\cos\left(tan^{-1}\left(\frac{a}{b}\right)\right)$$

Finding the coefficient of rolling resistance is outside the scope of this work. For theoretical model, I use a rolling resistance coefficient of 0.0032 (Groeskamp, 2017).

Weight

The most important force affecting the cyclist in this investigation is weight. On a flat plane, weight does not act as a resisting force on the cyclist, however, it is responsible for creating rolling resistance as is mentioned in the section above. Since this investigation explores how the hill gradient affects the velocity, weight is the biggest source of resisting force in this experiment. Weight can be both resolved to calculate the normal force and the resisting force. The equation for weight is given by the equation:

$$F_g = mg$$

and work needed to change the altitude is given by:

$$W_{PE} = mga$$

where $a$ is change in altitude. Power needed to gain altitude is:

$$P_{PE} = \frac{W_{PE}}{t} = mg\frac{a}{t}$$

Where $t$ is time needed to gain altitude $a$ and $\frac{a}{t}$ is vertical velocity (vertical component of $v_g$), which can be expressed as a function of $v_g$ as follows:

$$\frac{a}{t} = v_g \sin\left(\tan^{-1}\left(\frac{a}{b}\right)\right)$$

Power needed to gain altitude as a function of ground velocity is then:

$$P_{PE} = mgv_g \sin\,(tan^{-1}\left(\frac{a}{b}\right))$$

At zero wind, the power needed to ride uphill at a given speed is:

$$P_{total} = E_C^{-1}(\frac{1}{2}\rho C_d A v_g^3 + C_{RR} mgv_g \cos\,(tan^{-1}\left(\frac{a}{b}\right)) + mgv_g \sin\,(tan^{-1}\left(\frac{a}{b}\right)))$$

This formula enables us to calculate the total Power required to keep moving at certain speed. The formula misses one important element, which is the Power needed for acceleration. Besides that, there are few other sources of Power, which are not part of the formula, but they will not be mentioned in this investigation together with the power to accelerate, since it is such a broad aspect that doesn't fit the scope of this essay.

## HYPOTHESIS

It is predicted that an increase in the hill gradient will lead into a decrease in the velocity of the cyclist. This assumption is based on the fact that as the hill gradient increases, the cyclist opposes a greater resolved force of gravity. The cyclist encounters the composition of the three forces. Each force has a different variation with velocity. The relationship between air drag and velocity is relative, as the velocity of the cyclist increases, the air-drag increases exponentially. On a flat plane, the rolling resistance is directly proportional to velocity, but taking into account that the hill gradient changes, the normal force has to be determined by resolving the acceleration due to gravity. As the rolling resistance does not have a great influence on the results, it won't affect the behaviour of the relationship between the hill gradient and the velocity significantly. When dealing with the weight, in the case of my experiment, the equation composes of two constant values (mass and gravitational acceleration). If the equation consisted only of these two elements, then the relationship between the hill gradient and velocity could be linear, since weight is the biggest source of resisting force. Nevertheless, the force opposing the cyclist is resolved using the sine function. Since the research question investigates the hill gradient instead of an angle, we need to use the inverse tan function to find the angle as shown in the framework. Therefore, I expect the relationship between the hill gradient and velocity to be similar to the function:

$$\sin\left(tan^{-1}\left(\frac{a}{b}\right)\right).$$

## THE EXPERIMENT SETUP

In order to study the effect of changing gradient on velocity, it is needed to either (i) first to know or measure gradient of various roads or (ii) ride roads with various (or changing) gradients and later select sections with desired average gradients using an application. The second option will be carried out, since it is easier to do. The bicycle used is a S-Works Tarmac equipped with one-sided power meter Stages and GPS cycling computer Garmin Edge 820. The bicycle was thoroughly cleaned, the drivetrain and derailleurs were properly lubricated, and the tyres were adjusted to have a pressure of 100 PSI (6.8 ATM). The weight of the bicycle was kept constant by not carrying water bottle nor any other carry-ons during the experiment. Before each ride, the weather and wind forecasts were checked, in order to avoid windy and moist conditions. During each ride, it was attempted to keep the same position and as well as keeping the power output at 165 W.

**Figure 1.5 – Picture of the same model of bicycle used in the experiment, (Bike chaser, 2018)**

**Figure 1.6 – Picture of the Stages Power Meter.**


**Figure 1.7 – Picture of the Stages Power meter and Garmin 820**

## INDEPENDENT VARIABLE

The independent variable in this experiment is the Hill gradient. The Theoretical model assumes constant gradient over the entire section, which is impossible to achieve in real life conditions. Instead, I am using average gradient defined as altitude gained over horizontal distance covered. A GPS computer measured both altitude gain and horizontal distance covered. If I assume that my position was identified with accuracy ranging in meters, this may create a significant error for short rides of less than a kilometre. Some of the mistakes are corrected by map matching algorithms, which match the measured GPS position to a position on the map. After uploading data to Strava, I carefully chose road segments with gradients ranging from 1 to 16 percent with preferably steady incline.

## DEPENDENT VARIABLE

The variable that was measured is velocity. Again, it will be measured using a GPS device and then averaged to have a mean velocity. Since no road-section inclines steadily, velocity changes when power output is kept constant. Such changes in velocities result in exponential increase in air drag power. At low velocities, this effect is marginal.

## CONTROLLED VARIABLES

- Mass of bike and accessories – This was done by using the same bicycle and accessories during the experiment. (easy to control)
- Drivetrain efficiency – This was done by cleaning and lubricating the bicycle before the experiment. Additionally, during the experiment, the cyclist rode only on asphalt roads to avoid having the dirt and debris from the road to get stuck in the drivetrain, which could eventually decrease the drivetrain efficiency. (easy to control)
- Rider position – During the experiment, the cyclist was riding in the same position in order to keep the air drag constant. (easy to control)
- Tyre pressure – This was done by inflating both tyres to the same pressure level. (easy to control)
- Wind speed and direction – This variable could not have been controlled. Therefore, in order to minimise the effect of wind speed and direction, non-windy days were chosen for the experiment. (moderate to control)
- Road surface – This was done by riding on the roads with similar surface, therefore keeping the coefficient of rolling resistance similar. (moderate to control)
- Road condition (wet or dry) – This was done by choosing non-rainy days for experiment and therefore keeping the road condition dry. (moderate to control)

From all controlled variables, power output proved the most difficult to control and keep constant throughout the investigation. This was because the figure shown on my computer was constantly changing. I could only check the average power output once I uploaded the riding data to Strava. I was relatively successful as my average power output for selected segments ranged from 162 to 171 W. Accuracy of power meter measurement may be an issue especially at high gradients and low speeds. It is important to maintain cadence of 60 to 90 revolutions per minute and pay attention to steady and smooth pedaling technique where power applied by both legs along the entire revolution.

## DATA ANALYSIS

| Segment Name | Time (s) | Distance (m) | El. Gain (m) | Hor. Dist (m) | Gradient (%) | Velocity (m/s) | Power (W) |
|---|---|---|---|---|---|---|---|
| Kamzik-strma | 331 | 480 | 72 | 475 | 15,2 | 1,45 | 171 |
| Kacin-stojka | 199 | 330 | 39 | 328 | 11,9 | 1,66 | 165 |
| To Radio Tower | 697 | 1180 | 138 | 1172 | 11,8 | 1,69 | 166 |
| U-cesta | 464 | 800 | 90 | 795 | 11,3 | 1,72 | 168 |
| Hlboka cesta | 264 | 460 | 48 | 457 | 10,5 | 1,74 | 169 |
| Hlboka-Funus | 286 | 570 | 53 | 568 | 9,3 | 1,99 | 168 |
| letmo tabula rampa | 898 | 1830 | 166 | 1822 | 9,1 | 2,04 | 165 |
| Lanovka to Kamzik | 883 | 1980 | 160 | 1974 | 8,1 | 2,24 | 164 |
| Hlboka Kick | 46 | 90 | 7 | 90 | 7,8 | 1,96 | 167 |
| Evening Ride 1 | 932 | 2190 | 163 | 2184 | 7,5 | 2,35 | 163 |
| naKmazikOdbojara | 466 | 1070 | 78 | 1067 | 7,3 | 2,30 | 162 |
| Karpatska Kamzik | 1321 | 3470 | 238 | 3462 | 6,9 | 2,63 | 165 |
| Ricardo Slamka | 683 | 2020 | 128 | 2016 | 6,3 | 2,96 | 164 |
| Posledna rampa | 146 | 480 | 28 | 479 | 5,8 | 3,29 | 162 |
| ZS-Kacin | 942 | 3690 | 156 | 3687 | 4,2 | 3,92 | 162 |
| Tretia-Jeseniova | 121 | 470 | 19 | 470 | 4,0 | 3,88 | 166 |
| Udolim Bystricky | 593 | 2680 | 93 | 2678 | 3,5 | 4,52 | 166 |
| Pezinská Baba | 1751 | 8210 | 283 | 8205 | 3,4 | 4,69 | 164 |

**Figure 1.8 – shows the table that summarizes the segmented rides and the data that have been recorded for the experiment.**

Chart below shows the differences between average power output measured by the power meter and average power output predicted by the model. Measured power has an error of 3,31% consisting of 3% power meter accuracy claimed by the manufacturer and rounding error as a percentage of lowest measured power output (0.5/162W).

| Assumptions | Amount | Unit |
|---|---|---|
| Weight (bike+rider) | 83 | kg |
| Gravity | 9,81 | m/s^2 |
| Air density | 1,2 | kg/m^3 |
| CdA | 0,3 | m^2 |
| Crr | 0,0032 | |
| Ec | 0,97 | |

**Figure 1.9 – shows a table with values for different constants from the experiment, that are used for calculations.**

| Gradient (%) | Measured Power (W) | Measured Velocity (m/s) | Expected velocity (m/s) at 165 W | Deviation (%) |
|---|---|---|---|---|
| 15,2 | 171 | 1,45 | 1,28 | -13,28 |
| 11,9 | 165 | 1,66 | 1,61 | -3,11 |
| 11,8 | 166 | 1,69 | 1,63 | -3,68 |
| 11,3 | 168 | 1,72 | 1,69 | -1,78 |
| 10,5 | 169 | 1,74 | 1,81 | 3,87 |
| 9,3 | 168 | 1,99 | 2,03 | 1,97 |
| 9,1 | 165 | 2,04 | 2,08 | 1,92 |
| 8,1 | 164 | 2,24 | 2,31 | 3,03 |
| 7,8 | 167 | 1,96 | 2,39 | 17,99 |
| 7,5 | 163 | 2,35 | 2,48 | 5,24 |
| 7,3 | 162 | 2,30 | 2,54 | 9,45 |
| 6,9 | 165 | 2,63 | 2,67 | 1,50 |
| 6,3 | 164 | 2,96 | 2,89 | -2,42 |
| 5,8 | 162 | 3,29 | 3,11 | -5,79 |
| 4,2 | 162 | 3,92 | 4,03 | 2,73 |
| 4 | 166 | 3,88 | 4,18 | 7,18 |
| 3,5 | 166 | 4,52 | 4,59 | 1,53 |
| 3,4 | 164 | 4,69 | 4,68 | -0,21 |

**Figure 2.0 – shows a table of the results of measured velocities at different hill gradients.**

The values in **Figure 2.0** shows the comparison and deviation of theoretical and actual results of velocities in the experiment, with the theoretical ones being calculated by the power equation shown below, assuming that the power has stayed constant throughout the experiment at 165W, without deviations.

The Deviation of the actual recorded velocity from the expected one shows, how much the expected values of velocity calculated by the theoretical model differ from the values of measured velocity. For the calculation of expected velocity, the formula from the theoretical framework was used.

$$P_{total} = E_C^{-1}\left(\frac{1}{2}\rho C_d A v_g^3 + C_{RR} m g v_g \cos\left(tan^{-1}\left(\frac{a}{b}\right)\right) + m g v_g \sin\left(tan^{-1}\left(\frac{a}{b}\right)\right)\right)$$

The values from the table 1.2 were substituted in this equation. The expected velocity was calculated in the excel, because this equation is too difficult to be expressed in the form of velocity.

The following graph (Figure 2.2) shows relationship between hill gradient and velocity at constant power output. Blue dots represent measured power output and velocities (note that power output fluctuates between 162 and 171W). Orange dots represent velocities for same gradient calculated by model at exactly 165W. Velocity is inversely related to hill gradient.

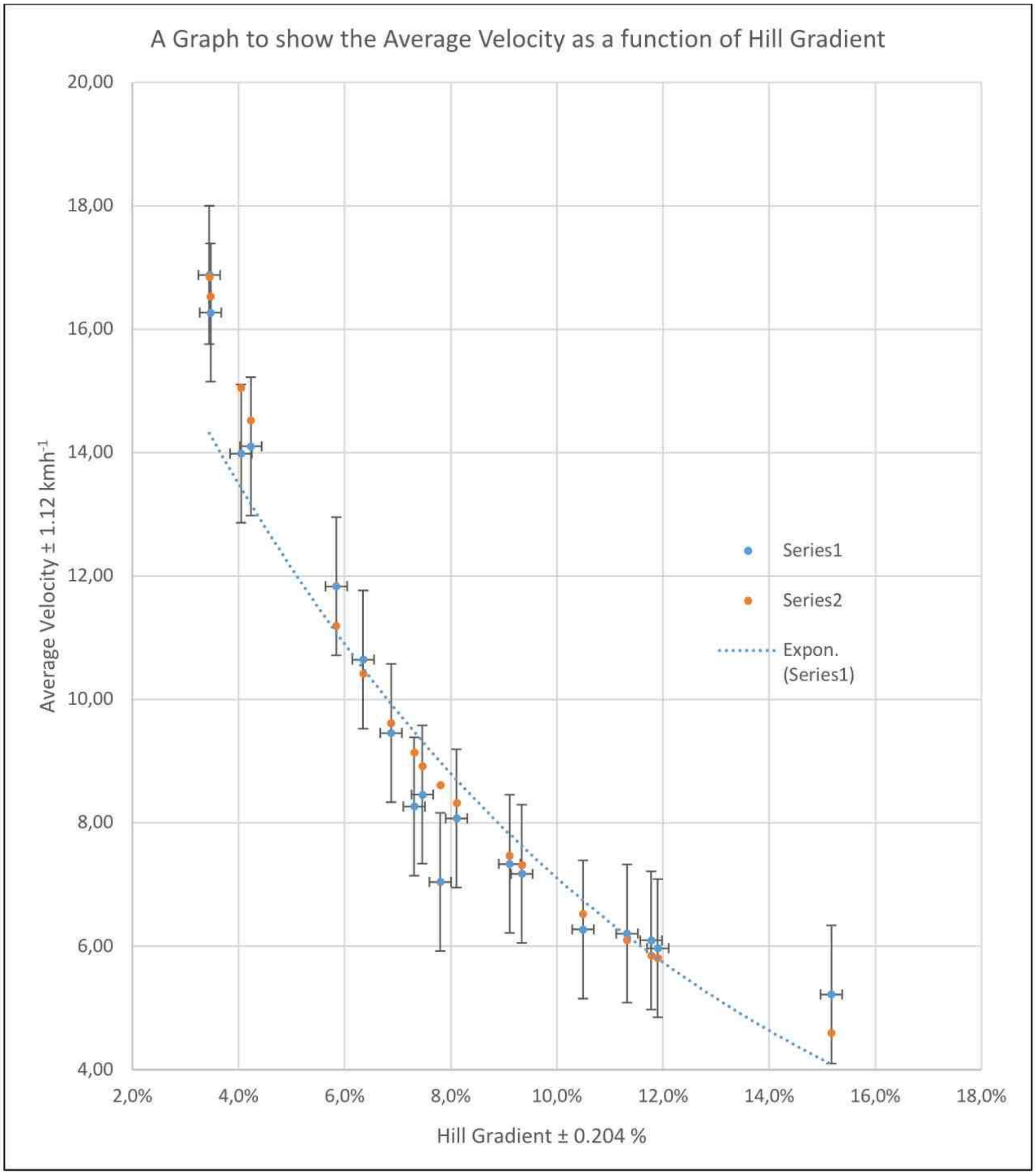


**Figure 2.2 – Graph of velocity as a function of hill gradient with error bars.**

The graph in **Figure 2.2** shows how velocity varies with the hill gradient. For this graph, the velocity is given in km $h^{-1}$. The relationship between the two variables is indeed exponentially declining.

## ERRORS AND UNCERTAINTIES

Power Meter Accuracy

Manufacturer claims that, the inaccuracy of the power measurement is ± 3 percent due to technical limitations (Stages cycling, 2018). Many discussion forums compare various models of power meters and find significant differences. A percentage error of $\pm 3.62$ % was given to Power, because Power also had a random error of 0.62% due to the limitation of reading, which was obtained by dividing the smallest value of Power shown on the reading by the smallest value of Power: $\frac{1}{162} \times 100 = 0.62$ %. The purpose of this table is to show, how much power would be required to have the same average velocity for particular gradients as in my experiment. This table was calculated in Excel using the theoretical model.

| PM Range 3,62 % | PM Range -3,62 % | Expected Power (W) | Gradient (%) |
|---|---|---|---|
| 177,2 | 164,8 | 187,0 | 15,2% |
| 171,0 | 159,0 | 169,8 | 11,9% |
| 172,0 | 160,0 | 171,6 | 11,8% |
| 174,1 | 161,9 | 168,4 | 11,3% |
| 175,1 | 162,9 | 158,3 | 10,5% |
| 174,1 | 161,9 | 162,4 | 9,3% |
| 171,0 | 159,0 | 162,2 | 9,1% |
| 169,9 | 158,1 | 160,2 | 8,1% |
| 173,0 | 161,0 | 134,4 | 7,8% |
| 168,9 | 157,1 | 155,5 | 7,5% |
| 167,9 | 156,1 | 148,9 | 7,3% |
| 171,0 | 159,0 | 161,6 | 6,9% |
| 169,9 | 158,1 | 170,0 | 6,3% |
| 167,9 | 156,1 | 176,4 | 5,8% |
| 167,9 | 156,1 | 160,7 | 4,2% |
| 172,0 | 160,0 | 153,1 | 4,0% |
| 172,0 | 160,0 | 160,9 | 3,5% |
| 169,9 | 158,1 | 167,4 | 3,4% |

**Figure 2.3 – table shows the error in the power output, and the expected power outputs for the values of velocity that were recorded in the experiment calculated by the theoretical model.**

GPS Computer Accuracy

I relied for measurement of time, distance, elevation gain and velocity on a GPS based cycling computer. Accuracy of such measurements is simply a function of how accurate the GPS device identifies my position. Actual performance exceeds the specification significantly. On May 11, 2016, the global average URE was ≤0.715 m (2.3 ft.), 95% of the time." Manufacturer claims an accuracy of two to three meters for horizontal and vertical identification of position. Such uncertainty is responsible for significant errors in distance, velocity and altitude gain for short riding segments. For segments in order of kilometers, this uncertainty is rather marginal.

The difference between measured and expected power output correlates strongly with length of segment as shown in the following chart. The shorter the segment the larger the error.

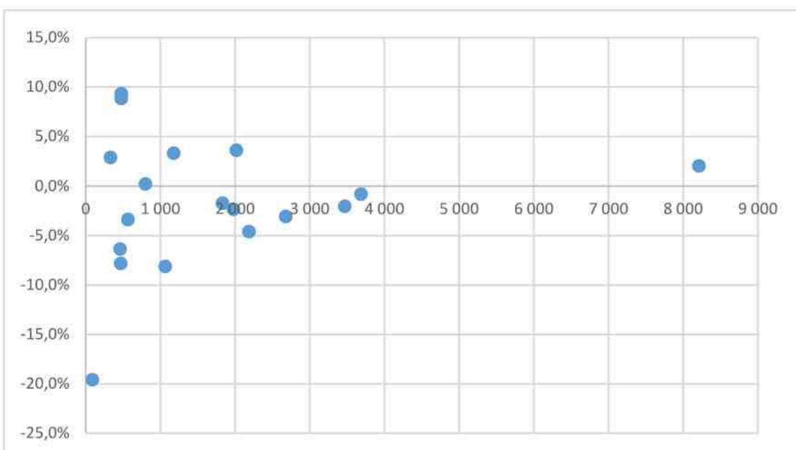


**Figure 2.1 – Graph of the distance of the rides and their inaccuracy compared to the theoretical model.**

Large errors for shorter segments maybe attributable to GPS inaccuracy in identifying correctly my position at the beginning and at the end of each segment. This phenomenon correctly corresponds the GPS inaccuracy. Its error is said to be is 2 or 3 metres for horizontal or vertical distance independent from the distance of the ride. This would explain, why the shorter segments are much more inaccurate.

Rider and Bike Mass

Mass is a linear parameter in two of three components of the power equation – rolling resistance and potential energy. Mass does not affect air drag directly, but it is correlated with CdA parameter, where A is a cross-sectional area of the rider and bike perpendicular to wind direction. Larger rider must face higher air resistance than smaller riders. The relationship of mass to cross sectional area differs for various shapes. For cube, the smallest cross-sectional area is equal to area of its side. Growth in volume of a cube by a factor of g results in growth in cross-sectional area of $g^{2/3}$. For sphere, it would be 0.25 $g^{2/3}$. Although mass and CdA are strongly correlated, I will look at sensitivity of error in CdA separately, as its impact is limited at low velocities of my experiment.

A 5% error in mass of rider and bike (4.15kg) would result in 4.98% to 4.43% change in total power required, providing CdA remains the same.

## CALCULATION OF UNCERTAINTIES

*Calculating the uncertainty for the velocity and the hill gradient*

1st Percentage error in Distance = $\frac{1}{90} \times 100 = 1.11$ %

2nd Percentage error in Distance = $\frac{3}{90} \times 100 = 3.33$ %

Total Percentage error in Distance = $1.11 + 3.33 = 4.44$ %

Percentage error in Time = $\frac{1}{46} \times 100 = 2.17$ %

Percentage error in Velocity = Percentage error in Distance + Percentage error in time

$$= 4.44 + 2.17 = 6.63 \%$$

Uncertainty in Velocity = $4.69 \times 0.0663 = 0.311$ $ms^{-1}$

The uncertainty in the velocity is $\pm$ 0.311 $ms^{-1}$.

The Garmin device calculates the velocity by the formula: $\frac{Distance}{Time}$. Nevertheless, the device also shows the value of velocity, but due to the fact that the calculated error of velocity would be in my opinion too inaccurate, I decided to calculate both the error in distance and error ir time to get the error in velocity by adding these two. The distance calculated has two random errors. The first error is a digital error that is due to the limitation of the reading on the computer. And the second error is due to the inaccuracy of GPS. I calculated this error based on the assumption that the inaccuracy of GPS positioning would be 3 meters. The time displayed on the computer has a digital error of 1 second, because that is the smallest unit shown.

Percentage error in hill gradient = Percentage error in 1st + Percentage error in 2nd

= 0.174 + 0.0433 = 0.217 %

The hill gradient, also recorded on the Garmin, is calculated by the formula:

$$\frac{El.\ Gain}{Horizontal\ Distance}$$

Similarly, to the calculation of the error in the velocity, I will calculate the error in hill gradient by adding the error in elevation gain and the error in horizontal distance.

Percentage error in El. Gain = $\frac{2}{53} \times 100 = 3.77$ %

Percentage error in Horizontal Distance = $\frac{2}{90} \times 100 = 2.22$ %

Percentage error in Hill Gradient = Percentage error in El. Gain + Percentage error in Horizontal Distance

Percentage error in Hill Gradient = $3.77 + 2.22 = 5.99$ %

Uncertainty in Hill Gradient = $3.4 \times 0.0599 = 0.204$ %

The uncertainty in the hill gradient is $\pm$ 0.204 %.

## CONCLUSION

In this essay, I investigated the relationship between velocity and gradient in cycling uphill, where air drag had a limited effect due to low velocities. Some of the findings contradicted my intuitive understanding, but the results were supported by the researcher hypothesis. It was predicted that an increase in the hill gradient will lead into a decrease in the velocity of the cyclist. The hypothesis was correct, and the second aspect of the hypothesis, the relationship, wasn't exactly fitting the assumptions made. Even though the relationship between the two variables was similar in shape to the graph of $\sin\left(tan^{-1}\left(\frac{a}{b}\right)\right)$, the two relationships had a different behavior. The relationship was found to be an exponential decline, as the velocity decreases.

## EVALUATION

In the experiment, there were no systematic errors present. However, there were some random errors that had to be dealt with. One of the random errors was in the velocity, which was significant in affecting the results, so it needed a fitting uncertainty to compensate it. It was calculated by adding the errors in distance and time. Both of these quantities had a random error in the device reading. All of the distances had a random error due to the GPS positioning inaccuracy, which was either 2 or 3 meters from the smallest value, this random error was again very influential over the results, which is why it was given the uncertainty. The error in elevation gain was the only exception, because the percentage error was created not from the smallest value, but a medium one. This decision was made, because the smallest value was too small, that it would create a very big error. The error in hill gradient was also very significant to impact the outcome of the results and it was calculated by adding the errors in elevation gain and horizontal distance. I consider that the values of my errors correctly represent the actual divergence of the experiment.

The controlled variable Power was a big source of error, which was only discussed in this essay, but was not taken into account to affect the results. This error was not significant, since the values of average power output were slightly different to the requisite value of 165 Watts. However, due to the fact that, power output could not have been kept constant during the experiment, it was increasing and decreasing randomly and therefore some energy was lost to acceleration.

Another random error that is worth mentioning but does not play a big role in affecting the experiment is caused by cadence. When a rider has a high cadence, some energy might be lost to the bicycle system. If the same cyclist rode at the same power output, but different cadency, his velocity would be higher with smaller cadency. There many things that could be improved. At first, power is not generally a good choice of controlled variable. If this could not be changed, a better power meter would improve the results. Besides that, it is important to choose accurate equipment for experiment. To improve the experiment even further, longer riding segments would reduce measurement errors significantly. Using multiple measuring devices (e.g. two power meters) or different measurement techniques (GPS and traditional speedometer) would help as well.

A limitation of this investigation was that the representation of the hill gradient wasn't correlational, but the percentages were scattered in a random order of increasing range. For future investigations, there should be specific segments chosen of correlational gradients. One of the surprises faces in the experiment was how much an aerodynamic setup reduces air drag and therefore saves power or allows higher velocity at same power output. This study did not focus on researching air-drag due to the limiting word count, however for future studies it should be taken into consideration for more a more detailed and broader research.

An interesting topic for future could be why heavier riders such as Peter Sagan cannot keep pace of pure climbers in high mountains or how does the power-weight ratio determine the type of the cyclist (sprinter, climber…).

# APPENDIX 1

| Date | Segment Name | Time | Distance | El. Gain | Hor. Dist. | Gradient | Velocity | mPOWER | ePE | eDRAG | eRR | ePOWER | Pct Diff |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 12.09.18 | Kamzik-strma | 331 | 480 | 72 | 475 | 15,2% | 5,22 | 171 | 183 | 1 | 4 | 187 | 9,4% |
| 12.09.18 | Kacin-stojka | 199 | 330 | 39 | 328 | 11,9% | 5,97 | 165 | 165 | 1 | 4 | 170 | 2,9% |
| 12.09.18 | To Radio Tower | 697 | 1 180 | 138 | 1 172 | 11,8% | 6,09 | 166 | 166 | 1 | 5 | 172 | 3,4% |
| 12.09.18 | U-cesta | 464 | 800 | 90 | 795 | 11,3% | 6,21 | 168 | 163 | 1 | 5 | 168 | 0,2% |
| 12.09.18 | Hlboka cesta | 264 | 460 | 48 | 457 | 10,5% | 6,27 | 169 | 153 | 1 | 5 | 158 | -6,4% |
| 12.09.18 | Hlboka-Funus | 286 | 570 | 53 | 568 | 9,3% | 7,17 | 168 | 156 | 1 | 5 | 162 | -3,4% |
| 12.09.18 | letmo tabula rampa | 898 | 1 830 | 166 | 1 822 | 9,1% | 7,34 | 165 | 155 | 2 | 5 | 162 | -1,7% |
| 12.09.18 | Lanovka to Kamzik | 883 | 1 980 | 160 | 1 974 | 8,1% | 8,07 | 164 | 152 | 2 | 6 | 160 | -2,3% |
| 12.09.18 | Hlboka Kick | 46 | 90 | 7 | 90 | 7,8% | 7,04 | 167 | 128 | 1 | 5 | 134 | -19,5% |
| 12.09.18 | Evening Ride 1 | 932 | 2 190 | 163 | 2 184 | 7,5% | 8,46 | 163 | 147 | 2 | 6 | 156 | -4,6% |
| 12.09.18 | naKmazikOdbojara | 466 | 1 070 | 78 | 1 067 | 7,3% | 8,27 | 162 | 141 | 2 | 6 | 149 | -8,1% |
| 12.09.18 | Karpatska Kamzik | 1 321 | 3 470 | 238 | 3 462 | 6,9% | 9,46 | 165 | 151 | 3 | 7 | 162 | -2,0% |
| 12.09.18 | Ricardo Slamka | 683 | 2 020 | 128 | 2 016 | 6,3% | 10,65 | 164 | 157 | 5 | 8 | 170 | 3,7% |
| 12.09.18 | Posledna rampa | 146 | 480 | 28 | 479 | 5,8% | 11,84 | 162 | 161 | 7 | 9 | 176 | 8,9% |
| 12.09.18 | ZS-Kacin | 942 | 3 690 | 156 | 3 687 | 4,2% | 14,10 | 162 | 139 | 11 | 11 | 161 | -0,8% |
| 12.09.18 | Tretia-Jeseniova | 121 | 470 | 19 | 470 | 4,0% | 13,98 | 166 | 132 | 11 | 10 | 153 | -7,8% |
| 12.09.18 | Udolim Bystricky | 593 | 2 680 | 93 | 2 678 | 3,5% | 16,27 | 166 | 132 | 17 | 12 | 161 | -3,1% |
| 13.09.18 | Pezinská Baba | 1 751 | 8 210 | 283 | 8 205 | 3,4% | 16,88 | 164 | 136 | 19 | 13 | 167 | 2,1% |

Legend:
Segment Name - Name of segment as per Strava
Time - time recorded in seconds
Distance - distance in meters
El. Gain - elevation gained in meters
Hor. Dist. - horizontal distance covered in meters
Gradient - average segment gradient in pct calculated as El. Gain to Hor. Dist.
Velocity - average velocity in kilometers per hour
mPOWER - average power output measured by power meter
ePE - expected power needed to gain potential energy as per model
eDRAG - expected power needed to overcome air resistance
eRR - expected power needed to overcome rolling resistance
ePOWER - expected total power output as per model
Pct Diff - percentage difference between ePOWER and mPOWER

**Table 2.4 – Table of the rides and data of calculated with measured quantities.**

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# EE/RPPF

**For use from May/November 2018**



International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Candidate personal code:

## Extended essay - Reflections on planning and progress form

***Candidate:*** *This form is to be completed by the candidate during the course and completion of their EE. This document records reflections on your planning and progress, and the nature of your discussions with your supervisor. You must undertake three formal reflection sessions with your supervisor: The first formal reflection session should focus on your initial ideas and how you plan to undertake your research; the interim reflection session is once a significant amount of your research has been completed, and the final session will be in the form of a viva voce once you have completed and handed in your EE. This document acts as a record in supporting the authenticity of your work. The three reflections combined must amount to no more than 500 words.*

**The completion of this form is a mandatory requirement of the EE for first assessment May 2018. It must be submitted together with the completed EE for assessment under Criterion E.**

***Supervisor:*** *You must have three reflection sessions with each candidate, one early on in the process, an interim meeting and then the final viva voce. Other check-in sessions are permitted but do not need to be recorded on this sheet. After each reflection session candidates must record their reflections and as the supervisor you must sign and date this form.*

## First reflection session

Candidate comments:

My initial idea was to research the effect of the centre of gravity on velocity in surfing. At first, I researched many articles about science behind surfing. I was able to develop the theory and understood all the forces that acted on the surfer. Since I was very passionate about the topic, I spent a lot of time on the research. It was hard to work with the liquid drag in surfing, as it depends upon several factors and has a different effect at different velocities. When it came to devising an experiment, it was easy to come up with an aim, however I struggled to find material, which could simulate a similar environment to actual surfing. Later, I realised that I didn't spend enough time, when considering my topic and the experiment. The topic was really interesting, as I could find out, what is the critical position, at which your board dives in and what is the maximum velocity you can obtain, but the small scale of the experiment couldn't match the real life conditions and would give ambiguous results. This experience taught me not always to choose my favourable topic, but something that is feasible.

Date: May 8, 2018

Supervisor initials:

International Baccalaureate® Baccalauréat International Bachillerato Internacional

## Interim reflection

Candidate comments:

Before I started writing, I spent a lot of time just researching different studies and aspects of physics behind cycling. I have already done my experiment and collected secondary data that I can use to analyse my experiment. During my experiment, I faced many challenges that affected it's precision. Firstly, I had to find the right equipment. Earlier, I measured the variables by Garmin and Strava, but it was not accurate, so I obtained a special pedals with the ability of measuring the right quantities. The next challenge was to find a day with perfect conditions for my experiment (no wind). When I first started writing, I lacked structure and my writing was unconcise and did not really make sense. So I decided to make a plan consisting of scheme and a structure of ideas connecting together. I reckon that as I now have a good structure, I can slowly start writing. The next challenge is to write a big amount of work in a short time range.

Date: October 26, 2018

Supervisor initia

## Final reflection - Viva voce

Candidate comments:

My ideas were realised to a great extent, since I managed to answer all the questions that were related to my research question. My initial assumption that the weight has a linear relationship with velocity was later proven to be wrong. My results made me realise that the relationship cannot be linear since the resolved force of weight has a trigonometric component. I really enjoyed the experimental process and the most rewarding part was, when I got all the gradients represented by valid results. I consider the experiment as the biggest success, as my results corresponded the theory, given that I struggled to devise an experiment to keep my CV controlled. As I plan to go study physics, this experience helped me to develop my experimental skills that will be very useful in future.

Date: February 11, 2019

Supervisor initials: